Abstract. A model for allocation of vessels’ time budget under long-term freight contracts in the conditions of fuzzy uncertainty is developed, taking into account the fact that in a «free» from work time under these contracts, vessels can operate in an open freight market. Fuzzy uncertainty is manifested in the fact that the parameters of the transport process, technical and operational indicators of vessels’ performance and the volumes of transport work are presented in the form of maximum, minimum and most expected, which corresponds to the actual situation in the shipping management.

The proposed model allows to define: the number of vessels of a certain type, which should be taken in time charter lease to achieve specific goals and conditions; the allocation of the company’s own vessels and leased vessels under long-term contracts and, accordingly, the selection of contracts (as a result of optimization, individual contracts may be identified as not effective for the company); determination of the time budget shares of vessels (own and leased), within which they will carry out transportation under these contracts, as well as time budget shares, within which they will work on the free freight market. At the same time, established values of the variables ensure the maximization of operating profit, with the consideration of its minimum permissible boundary.

Technical and operational performance of the vessels and the characteristics of contracts and indicators characterizing the situation on the freight market are presented in the form of fuzzy numbers of a triangular type, which reflects the practical availability of information (minimum, maximum, and most possible).

The results obtained from the model take into account the uncertainty of the conditions formulated in the form of fuzzy numbers, describing the values of technical and operational indicators, as well as the characteristics of long-term contracts and the situations on the freight market. Practical use of the model allows shipowners to plan the work of vessels and evaluate the results of their work in the absence of complete information, based on analysts’ forecasts, presented in the form of fuzzy numbers.

Keywords: shipping, fuzzy sets, optimization, fleet operating, contracts of affreightment.
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МОДЕЛЮВАННЯ РОЗПОДІЛУ БЮДЖЕТУ ЧАСУ СУДЕН ПО ДОВГОСТРОКОВИМ ФРАХТОВИМ КОНТРАКТАМ В УМОВАХ НЕВИЗНАЧЕННОСТІ

Анотація. Розроблено модель розподілу бюджету часу суден за довгостроковими контрактами в умовах неймовірної невизначеності з урахуванням того факту, що в «вільний» від роботи час за цими контрактами судна можуть працювати на відрядному фрактовому ринку. Нечітка невизначеність проявляється в тому, що параметри транспортного процесу, технічні та експлуатаційні показники роботи суден і обсяги транспортної роботи представлені у вигляді максимальної, мінімальної і найбільш очікуваного значення, що відповідає реальній ситуації в судноцінстві. Запропонована модель дозволяє визначити: кількість суден нового типу, яке необхідно взяти в тайм-чартерну аренду для досягнення конкретних цілей при заданих умовах; розподіли власних та орендованих суден компанії за довгостроковими контрактами і, відповідно, вибір контрактів (в результаті оптимізації охорони контракти можуть бути визначені як неефективні); визначення терміну і бюджету часу суден (власних та орендованих), в рамках яких вони здійснюють перевезення за цими контрактами, а також часи бюджету часу, в межах яких вони будуть працювати на фрактовому ринку. Встановлені значення змінних забезпечують максимізацію операційного прибутку, з урахуванням її мінімально допустимої межі. Практичне використання моделі дозволяє судовладінцям планувати роботу суден і оцінювати результати їх роботи при відсутності повної інформації, зазначеної на прогнозах аналітиків, представлених у вигляді невідомих чисел.

Ключові слова: морські перевезення, нечіткі множини, оптимізація, оперування флотом, фрахтові контракти.
Introduction. Shipping is a type of activity that is characterized by a high level of commercial risks connected with the uncertainty of the freight market conditions on the one hand, and the uncertainty of parameters reflecting the production process of vessels, that is, the parameters of the voyage, on the other.

While making a decision about commercial operation of vessels, the shipowner must take into account these commercial risks, regardless of the level of decisions made – operational, medium-term or strategic.

Long-term freight contracts (Contracts of Affreightment) are a form of vessel operation. In [1] it is indicated that long-term freight contracts have certain specific features, including, in most cases, the absence of a clear indication of ports of call and the volume of cargo traffic. Also, during fulfillment of obligations to cargo owners under these contracts, it may be necessary to attract vessels on a time charter basis in a situation when their own vessels are engaged in the carriage of cargoes on the free freight market.

Thus, the shipowner must compare the performance of the vessels on long-term freight contracts and on the open freight market, provided that a mixed version of the vessels operation is possible – partly on obligations under long-term contracts, partly – on the open freight market. At the same time, the terms of contracts, including the volume of transported goods, must be fulfilled.

During evaluation of the effectiveness, the shipowner should take into account the uncertainty of the working conditions of the vessels, regardless of the form of their work – on the open freight market or within long-term freight contracts.

The main factor contributing to the uncertainty of conditions during estimation of the performance of vessels' work in the open freight market is the level of freight rates (whose volatility is known to be quite high) and the volume of demand. The main factors contributing to the uncertainty of conditions during estimation of the performance of vessels' work under long-term freight contracts are the volume of transport work (can be set as an interval according to [1; 2]) and production parameters, which are difficult to estimate with a high degree of certainty due to the uncertainty of ports of call in the majority of such contracts (according to [3; 4]).

Literature review and problem statement. Uncertainty of freight market has been considered at many papers (for examples [5-11]). A significant part of publications devoted to modeling of the distribution of the vessels' time budget is connected with the distribution on schemes or linear services. At the same time, the practical majority of existing models are based on a deterministic version of setting of vessels' working conditions, since they were developed [1]: 1) during the period of a planned economy (domestic scientific school); 2) for the situation of linear shipping with a focus on a clearly defined set of ports of call, a specific sche-ule and an average variant of vessels' loading capacity.

The uncertainty of the conditions in which certain processes are carried out may be described in various ways depending on the available information. In problems of allocation character with a horizon of a considered period of time of about a year, there are
three possible approaches to the mathematical description of uncertainty:

- **Probabilistic** – in this situation, the types and parameters of the laws of allocation of random values, reflecting the indicators and parameters of conditions, must be known;
- **Interval** – in this situation, the law of allocation is unknown, but the minimum and maximum of the possible values of indicators and parameters of conditions can be estimated (for example, by an expert method);
- **Fuzzy** – a variant in which the previous situation occurs, but it is possible to evaluate not only the minimum and maximum values, but also to describe the behavior of intermediate values in the form of membership functions.

Probabilistic models for solving problems related to the allocation of vessels are usually used for long-term (more than one year) planning. In particular, in [9] the problem of allocation of vessels between working regions and between two variants of commercial operation options (voyage charter and time charter) was considered, taking into account the probabilistic nature of freight rates (the results of statistical studies that justified the use of the normal freight rates were used as a base).

It should be noted that the information available to the shipowner does not always allow the use of mathematical statistics methods for the further use of probabilistic methods. Thus, the practical use of probabilistic approaches for accounting of uncertainty is rather difficult in the most cases.

In [1], was proposed a model for allocation of vessels' time budget between long-term contracts and work in the open freight market. In order to take into account the uncertainty of the working conditions of vessels, an interval description of the relevant parameters and indicators was used.

However, with the possibility of operating with a large amount of information and, in particular, the possibility of estimating not only the maximum and minimum, but also the most expected value of a parameter or indicator, fuzzy sets theory can be used, which, unlike the interval description of uncertainty, will give more reasonable result, taking into account the operation of more structured information.

Thus, this research relies on a substantive level on the results presented in [1]. As a theoretical basis of using the theory of fuzzy sets in modeling problems of resource allocation, were used the results presented in [10; 11].

**The aim of the study.** The aim of this study is a development and practical testing of a model for allocation of vessels' time budget under long-term freight contracts, taking into account the uncertainty of their working conditions, described in terms of fuzzy sets.

**Results of modeling.** The structure of the proposed model reflects the purpose and basic limitations connected with the results of vessels' work for specific company question during planned period.

The main purpose of the allocation of vessels by type of work in the considered time period is to maximize profits, the structure of which should include:

- revenues from two types of vessels operations – on long-term freight contracts and on the open freight market;
- operating costs;
the cost of renting vessels for the case of using time-charter vessels.

The limitations of the model reflect next conditions:
• in transportation volumes under long-term freight contracts;
• in traffic volumes on the open freight market (in a given region);
• on time budget of own and rented vessels;
• by the number of leased vessels.

At the same time, it is assumed that in the open freight market there is no allocation of vessels for specific destinations and the vessels operate according to the classical tramp scheme. Therefore, during the estimation of the performance of vessels’ work in the open freight market, the averaged (for the region and for a given type of vessels) time-charter equivalent is taken as the base, and not the freight rates. The structure of the averaged time-charter equivalent takes into account [12] freight rates for the transportation routes within the region, operating costs, and the duration of voyages for the possible directions of transportation.

The assumption of the model is to consider only one region for the operation of vessels. A natural development of the model can be the consideration of the set of geographic regions of the freight market.

As it was determined above, the uncertainty of the conditions and results of the vessels’ work is given in the form of fuzzy sets. Following the results of [11-14], the most practical option is to use fuzzy triangular numbers for specified purposes – specific fuzzy sets whose structure corresponds to a three-level estimation of the values of parameters and indicators – maximum, minimum, and the most expected ones. Triangular fuzzy numbers have the form $\hat{A} = \langle a_1, a_2, a_3 \rangle$, and their membership function has the form

$$\mu_A(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, x \in [a_1, a_2], \\ \frac{x - a_2}{a_3 - a_2}, x \in [a_2, a_3], \\ 0, x \notin [a_1, a_3] \cup [a_2, a_3]. \end{cases}$$ (1)

At the same time the condition $a_1 \leq a_2 \leq a_3$ is fulfilled.

Let’s introduce the indications:
• $i = 1, m$ – indices of vessels of the company.
• The work of each vessel is characterized by $\tilde{R}_i = \langle r_{i1}, r_{i2}, r_{i3} \rangle$ – the daily rate of constant costs, dollars/day;
• $\overline{TCE}_i = \langle \overline{TCE}_{i1}, \overline{TCE}_{i2}, \overline{TCE}_{i3} \rangle$ – time-charter equivalent (during the work in the open freight market), dollars/day.

We also take into account the fact that in order to implement its plans for fulfillment of long-term freight contracts or working in the open freight market, a company can replenish its fleet based on the basis of time charter leases at rates $r_{k}^{t-ch}, k = 1, K$, where $k$ – is a vessel (prototype), $K$ – is a set of considered variants of vessels. At the same time, the work of rented vessels is characterized by the value of the time-charter equivalent

$$\overline{TCE}_k = \langle \overline{TCE}_{k1}, \overline{TCE}_{k2}, \overline{TCE}_{k3} \rangle$$
$I_s$ — coefficient that corrects considered period of time $T$, taking into account commercially unused time, that is, the time when vessels do not perform transportation (for example, unscheduled repairs, bad weather, etc.).

At the same time, condition $0 < I_s < 1$ is also fulfilled.

Let’s set $j = \overline{1,n}$ as indices of considered long-term freight contracts. Considered period of long-term contracts is $T$, days.

Each contract is characterized by the following technical and operational indicators

$\hat{P}_{ij} = \langle p_{i1}^j, p_{i2}^j, p_{i3}^j \rangle$ — carrying capacity of the $i$-th vessel during the operation under the $j$-th contract, thousand tons/year;

$\hat{P}_k = \langle p_{k1}, p_{k2}, p_{k3} \rangle$ — carrying capacity of the $i$-th vessel during the operation under the $j$-th contract, thousand tons/year.

Let’s introduce model variables:

$x_{ij}, 0 \leq x_{ij} \leq 1$ — the share of the time budget of the $i$-vessel under the $j$-th contract. Since not every vessel from the considered set can work under specific contract (for example, a cargo consignment does not fit in size for a given vessel), in such a situation, some $x_{ij}$ of the control parameters are taken exogenously as 0. This procedure will exclude from the further optimization «impossible» options for allocation of vessels under contracts [1];

$x_{kj}^{ch}, k = \overline{1,K}$ — the number of vessels of type $k$ leased under time charter;

$x_{kj}^{ch}, k = \overline{1,K}, j = \overline{1,n}$ — the share of the time budget of the $k$-th vessel under the $j$-th contract. Similarly to the arguments presented above, if a vessel $x_{kj}^{ch}$ cannot serve under the $j$-th contract, then the corresponding $x_{kj}^{ch}$ are set to 0 and are considered further as exogenous parameters.

According to the given designation of variables

$vessel under the $j$-th contract, dollars/day;

$\hat{p}_{ij}^l = \langle p_{i1}^l, p_{i2}^l, p_{i3}^l \rangle$ — carrying capacity of the $i$-th vessel during the operation under the $j$-th contract, thousand tons/year;
\[1 - \sum_{j=1}^{n} x_{ij}, i = 1, m \] - the share of the time budget of the i-th vessel in the open freight market;

\[1 - \sum_{j=1}^{n} x_{ij}, k = 1, K \] - the share of the time budget of the k-th vessel leased under time-charter on an open freight market;

\[x_{ij} - \sum_{j=1}^{n} x_{ij} \] - the share of the time budget of leased vessels out of long-term contracts - in the open freight market.

According to the model structure presented above, as an objective function reflecting the main goal of the company, we will use the maximization of profit from the work of vessels (own and leased) under long-term contracts and in the open freight market:

\[
\sum_{j=1}^{n} (f^{ij} \cdot (\sum_{k=1}^{K} \tilde{p}_{ik} \cdot x_{ij} + \sum_{k=1}^{K} \tilde{p}_{ik} \cdot x_{ij}^{ah}) - \sum_{j=1}^{n} \tilde{c}_{ij} \cdot x_{ij} - \tilde{R}) - \sum_{j=1}^{n} \tilde{c}_{ij} \cdot x_{ij}^{ah} + \sum_{j=1}^{n} I_{j} \cdot T \cdot \tilde{TC}E_{ij} \cdot \left(1 - \sum_{j=1}^{n} x_{ij}\right) + \sum_{j=1}^{n} I_{j} \cdot T \cdot \tilde{TC}E_{ij} \cdot (x_{ij}^{ah} - \sum_{j=1}^{n} x_{ij}^{ah}) \rightarrow \max_{x_{ij}, x_{ij}^{ah}}
\]

(2)

Considering the fuzzy nature of the indicators in (2) used as coefficients, the total profit value is also a fuzzy number \(\tilde{Pr}\).

Structure (2) corresponds to the structure of profits from the work of own and leased vessels under long-term contracts and in the open freight market:

\[\sum_{j=1}^{n} (\sum_{i=1}^{m} \tilde{p}_{ij} \cdot x_{ij} + \sum_{i=1}^{m} \tilde{p}_{ij} \cdot x_{ij}^{ah}) - \sum_{j=1}^{n} \tilde{c}_{ij} \cdot x_{ij} - \tilde{R}\] - amount of cargo transported under all long-term contracts by all vessels (own and leased);

\[\sum_{j=1}^{n} (f^{ij} \cdot (\sum_{k=1}^{K} \tilde{p}_{ik} \cdot x_{ij} + \sum_{k=1}^{K} \tilde{p}_{ik} \cdot x_{ij}^{ah}) - \sum_{j=1}^{n} \tilde{c}_{ij} \cdot x_{ij}^{ah} + \sum_{j=1}^{n} I_{j} \cdot T \cdot \tilde{TC}E_{ij} \cdot \left(1 - \sum_{j=1}^{n} x_{ij}\right) + \sum_{j=1}^{n} I_{j} \cdot T \cdot \tilde{TC}E_{ij} \cdot (x_{ij}^{ah} - \sum_{j=1}^{n} x_{ij}^{ah})\] - amount of freight from the work of vessels (own and leased) under contracts;

\[\sum_{j=1}^{n} (\tilde{p}_{ij} \cdot x_{ij} - \tilde{R})\] - operating expenses and fixed costs of the company's own vessels;

\[\sum_{j=1}^{n} \tilde{c}_{ij} \cdot x_{ij}^{ah}\] - operating costs of leased vessels;

\[\sum_{k=1}^{K} \tilde{c}_{ik} \cdot x_{ij}^{ah}\] - costs of leased vessels under time-charter;

\[\sum_{j=1}^{n} I_{j} \cdot T \cdot \tilde{TC}E_{ij} \cdot (x_{ij}^{ah} - \sum_{j=1}^{n} x_{ij}^{ah})\] - profit excluding fixed costs from the work of own vessels outside long-term contracts - in the open freight market;

\[1 - \sum_{j=1}^{n} x_{ij}\] - the share of the time budget of own vessels outside long-term contracts;

\[\sum_{j=1}^{n} (f^{ij} \cdot (\sum_{k=1}^{K} \tilde{p}_{ik} \cdot x_{ij} + \sum_{k=1}^{K} \tilde{p}_{ik} \cdot x_{ij}^{ah}) - \sum_{j=1}^{n} \tilde{c}_{ij} \cdot x_{ij}^{ah} + \sum_{j=1}^{n} I_{j} \cdot T \cdot \tilde{TC}E_{ij} \cdot (x_{ij}^{ah} - \sum_{j=1}^{n} x_{ij}^{ah}))\] - profit excluding fixed costs from the operation of rented vessels in the open freight market.

The limitations of the model correspond to the listed above conditions.
for contracts, vessels and the freight market:

in volume of transport work for each contract
in of the volume of transport work in the free freight market (in a given geographical region)

\[ \sum_{j=1}^{n} \hat{p}_j \cdot (1 - \sum_{j=1}^{n} x_{ij}) + \sum_{j=1}^{n} \hat{p}_j \cdot (1 - \sum_{j=1}^{n} x_{ij}) \subseteq \hat{Q} ; \]  

(4)

in the time budget of the company's own vessels

\[ \sum_{j=1}^{n} x_{ij} \leq 1, j = 1, \cdots, m ; \]  

(5)

in time budget of leased vessels

\[ \sum_{j=1}^{n} x_{ijk} \leq x_{ik}^{\text{char}} , k = 1, \cdots, K ; \]  

(6)

in the number of leased vessels

\[ x_{ik}^{\text{char}} \leq N_k , k = 1, \cdots, K . \]  

(7)

\[ N_k \] – the possible number of vessels for rent.

Restrictions in the possible values of the time budget allocation of own vessels

\[ 0 \leq x_{ij} \leq 1, i = 1, \cdots, m, j = 1, \cdots, n . \]  

(8)

\[ \sum_{j=1}^{n} (f_j \cdot (\sum_{j=1}^{n} \hat{p}_j \cdot x_{ij} + \sum_{k=1}^{K} \hat{p}_j \cdot x_{ijk}) - \sum_{j=1}^{n} (\overline{r}_j \cdot x_{ij} - \overline{R}) - \sum_{k=1}^{K} \overline{r}_j \cdot x_{ijk}) - \sum_{j=1}^{n} \overline{I}_j \cdot \overline{T} \cdot \overline{\text{C}} \cdot (1 - \sum_{j=1}^{n} x_{ij}) + \sum_{j=1}^{n} \overline{I}_j \cdot \overline{T} \cdot \overline{\text{C}} \cdot (x_{ijk} - \sum_{j=1}^{n} x_{ijk}) \) \supseteq \overline{\text{Pr}}^* \]  

(11)

It should be noted that during the formulating of model restrictions, those restrictions in which fuzzy numbers \((3), (4), (11)\) are to be compared, are written with the consideration of the rules of the fuzzy sets theory using appropriate signs.

The proposed model \((2)-(11)\) allows to define:

1) the number of vessels of a certain type, which should be taken in time charter lease to achieve specific goals and conditions;

2) the allocation of the company's own vessels and leased vessels under long-term contracts and, accordingly, the selection of contracts (as a result of optimization, individual contracts
may be identified as not effective for the company;
3) determination of the time budget shares of vessels (own and leased), within which they will carry out transportation under these contracts, as well as time budget shares, within which they will work on the free freight market. At the same time, established values of the variables ensure the maximization of operating profit, with the consideration of its minimum permissible boundary.

The results obtained from the model take into account the uncertainty of the conditions formulated in the form of fuzzy numbers, describing the values of technical and operational indicators, as well as the characteristics of long-term contracts and the situations on the freight market. To carry out calculations within proposed model, it is possible to use the methods that are described in detail in [15].

In particular, one of such approaches involves a sequence of the following steps:
1) the introduction of discrete \( \alpha \)-levels
   (usually \( \alpha \in \{0, 2; 0, 4; 0, 6; 0, 8; 1\} \));
2) transformation of the objective function and constraints to a clear view, taking into account the relevant rules and obtaining of optimization models with clear parameters, the number of models corresponds to the number of \( \alpha \) – levels;
3) obtaining optimal solutions for each model of \( \alpha \) – level and their transformation into a deterministic solution on the basis of the defuzzing procedure.

This method is tested in solving of practical problems and allows to obtain the required solution by using of standard applications for solving of optimization problems.

**Conclusion.** This research presents a model that allows, under conditions of uncertainty characterized by fuzzy sets, to allocate company’s own vessels and leased vessels under long-term freight contracts, taking into account the fact that during their free time under these contracts, vessels can operate on the open freight market.

Technical and operational performance of the vessels and the characteristics of contracts and indicators characterizing the situation on the freight market are presented in the form of fuzzy numbers of a triangular type, which reflects the practical availability of information (minimum, maximum, and most possible).

Practical use of the model allows shipowners to plan the work of vessels and evaluate the results of their work in the absence of complete information, based on analysts' forecasts, presented in the form of fuzzy numbers.

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